NorSIKT IP

Some notes on Time Mean Speed and Space Mean Speed
Summary
Vehicle speed and travel time are important variables when analyzing, for example, traffic flow, traffic safety and emissions from road traffic. Two measures of vehicle mean speed are central in such analyses; Time mean speed and Space mean speed. In this memorandum these concepts are explained, as well as various other terms relating to road traffic speed and travel time, such as point speed, travel speed, concentration and total vehicle mileage.

The research has been conducted within the “NorSIKT IP” project which aims at harmonisation of traffic parameters between the Nordic countries. The purpose is to create a foundation for the development of a shared comparable speed index for the Nordic countries.
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Some notes on Time Mean Speed and Space Mean Speed

Photo: Torbjørn Haugen

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Summary

Vehicle speed and travel time are important variables when analyzing, for example, traffic flow, traffic safety and emissions from road traffic. Two measures of vehicle mean speed are central in such analyses; Time mean speed and Space mean speed. In this memorandum these concepts are explained, as well as various other terms relating to road traffic speed and travel time, such as point speed, travel speed, concentration and total vehicle mileage.

The research has been conducted within the “NorSIKT IP” project which aims at harmonisation of traffic parameters between the Nordic countries. The purpose is to create a foundation for the development of a shared comparable speed index for the Nordic countries.

1. Background

NorSIKT (Nordic System for Intelligent Classification of Traffic) is a collaborative project between the road authorities in the Nordic countries (Sweden, Denmark, Iceland, Finland, the Faroe Islands and Norway). The project is funded through the NordFOU scheme by the Nordic road authorities.

The project has been running for several years with various subprojects. This memorandum is part of NorSIKT IP (NorSIKT Implementation) whose purpose is to implement pan-Nordic indexes. This memorandum explains various terms relating to speed and travel time, as a foundation for the development of a shared comparable speed index for the Nordic countries.

The report is organized in the following way. The basic speed terminology, Space Mean Speed and Time Mean Speed, is introduced in Section 2, illustrated by examples from real-world data (Section 3) and exercises (Section 4) on artificial data. In Section 5 a more theoretical approach is introduced. In Section 6 the advantages and disadvantages of Space Mean Speed and Time Mean Speed are summarized. Sections 1-4 and 7-8 is written by Haugen and Sections 5-6 jointly by Nyfjäll and Forsman. Haugen and Nyfjäll/Forsman have coordinated notation and terminology but otherwise worked independently.
2 Speed terminology

“Velocity” is an expression of how far an object travels in a given unit of time. The term “speed” is also commonly used. In international scientific literature, speed is represented by the letter \( v \) or \( u \). In this memorandum, we have chosen to use \( u \).

A common measure of speed is \( \text{km/h} \) (kilometres per hour) or \( \text{m/sec} \) (metres per second). The conversion factor from \( \text{km/h} \) to \( \text{m/sec} \) is 3.6 (1 hour is 3600 sec and 1 km are 1000 m).

Here are a couple of examples:

- \( 72 \text{ km/h} = 72/3.6 = 20 \text{ m/sec} \)
- \( 30 \text{ m/sec} = 30 \cdot 3.6 = 108 \text{ km/h} \)

A distinction is usually made between the speed of an individual vehicle and the average speed of multiple vehicles across a given time period.

When measuring the average speed of multiple vehicles, a distinction is also made between time mean speed (TMS) and space mean speed (SMS). Time mean speed is based on momentary snapshots, whereas space mean speed is calculated from time duration over a road section.

2.1 Point speed and Time mean speed

Point speed is the speed that can be measured at a given point on a road section. It is also called point speed if the speed is based on time duration over a short road section (a few metres).

The average speed of multiple vehicles passing a point is called time mean speed.

The time mean speed is calculated by adding all the point speeds up and dividing by the number of vehicles (i.e., the arithmetic mean of the vehicles point speeds):

\[
\bar{u}_p = \frac{u_{p1} + u_{p2} + \cdots + u_{pn}}{n} = \frac{\sum_{i=1}^{n} u_{pi}}{n}
\]

Where: \( \bar{u}_p \) = average point speed = time mean speed

\( u_{pi} \) = point speed of vehicle \( i \)

\( n \) = number of vehicles
2.2 Travel speed and Space mean speed

Travel speed is based on the time taken to travel a section of road and is defined as the length of a section divided by time duration across the section.

Travel speed of an individual vehicle is found by:

\[ u_{si} = \frac{d}{t_i} \]

Where: \( u_{si} \) = Travel speed of vehicle no. \( i \)

- \( d \) = Length of road section (distance)
- \( t_i \) = Travel time for vehicle no \( i \)

The average travel speed for a stream of traffic is called Space mean speed\(^1\). When calculating the space mean speed for a stream of traffic, the time duration is the central parameter. The easiest way to explain this is to first calculate the average time duration, then calculate the average travel speed by dividing the length of the road section by average time duration.

Formulae are given by:

\[ \bar{t} = \frac{t_1 + t_2 + \ldots + t_n}{n} = \frac{1}{n} \sum_{i=1}^{n} t_i \]

\[ \bar{u}_s = \frac{d}{\bar{t}} = \frac{d}{n} \sum_{i=1}^{n} t_i \]

Where:
- \( \bar{t} \) = Average travel time on road section
- \( \bar{u}_s \) = Space mean speed
- \( d \) = Length of road section
- \( t_i \) = Travel time for vehicle \( i \)
- \( n \) = Number of vehicles

\(^1\)The most common term is “space mean speed”, but “section speed” is also used.
Using the formula above, we can derive an expression for calculating the space mean speed using measurements of speed of individual vehicles. In many cases, this formula can be easier to use, but is not as easy to understand as the formula-based time duration. The result will however be the same.

\[ \overline{u}_s = \frac{d}{\frac{1}{n} \sum_{i=1}^{n} t_i} = \frac{d}{\frac{1}{n} \sum_{i=1}^{n} \frac{1}{u_{si}}} = \frac{1}{\frac{1}{n} \sum_{i=1}^{n} \frac{1}{u_{si}}} \]

Where:  
- \( \overline{u}_s \) = Space mean speed  
- \( d \) = Length of road section  
- \( u_{si} \) = Travel speed of vehicle \( i \)  
- \( t_i \) = Travel time for vehicle \( i \)  
- \( n \) = Number of vehicles

Note that \( \overline{u}_s \) is the harmonic mean of speeds for individual vehicles \( u_{si} \)

### 2.3 Relationship between time mean speed and space mean speed

The time mean speed for a stream of traffic will always be greater than or equal to the space mean speed, assuming the speed remains constant across the road section. Thus, if a road section has a constant speed level, the time mean speed will be greater than the space mean speed. This is because when calculating space mean speed, slow-moving vehicles are given more weight than in the calculation for time mean speed. In the calculation of space mean speed, the time duration is weighted rather than the speed. How great the difference between the two speeds will be depends on the spread of the individual vehicles, among other things.

The formula for the relationship was developed by Wardrop (1952):

\[ \overline{u}_p = \overline{u}_s + \frac{\sigma_{u_{si}}^2}{\overline{u}_s} \]

Where:  
- \( \overline{u}_p \) = Time mean speed  
- \( \overline{u}_s \) = Space mean speed  
- \( \sigma_{u_{si}}^2 \) = Variance (spread) of travel speeds for vehicle \( i \)

The difference between time mean speed and space mean speed across a road section with constant speeds will also depend on the spread of the speeds. The time mean speed and space mean speed will only be equal if the variance is zero, i.e. if all vehicles are travelling at a constant and equal speed.
3 Real-world examples

Here we have a real-world example where travel speed was measured over several hundred metres, and where time mean speed was measured inside the road section. The geometry of the road section is such that the individual speeds tend to be similar.

The first known experiment was carried out in Chicago in 1967 (FHA, 1997). Using a regression analysis, the following equation was devised:

$$\bar{u}_s = 1.026 \cdot \bar{u}_p - 3.042$$

<table>
<thead>
<tr>
<th>Equation Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{u}_s$</td>
</tr>
<tr>
<td>$\bar{u}_p$</td>
</tr>
</tbody>
</table>

The unit in the equation is [km/h]

In 1995, similar recordings were made on the E18 in Vestfold, Norway (Haugen, 1995 and 1996), see figure 1 and 2. Over a road section of 281 metres, time duration was measured by detecting vehicles with electronic toll tags, and space mean speed was calculated. Speed was also measured at a point along the road section using induction loops. The data was aggregated into a 5-minute average, and a regression analysis carried out. The result was:

$$\bar{u}_s = 1.050 \cdot \bar{u}_p - 4.87$$

<table>
<thead>
<tr>
<th>Equation Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{u}_s$</td>
</tr>
<tr>
<td>$\bar{u}_p$</td>
</tr>
</tbody>
</table>

The unit in the equation is [km/h]
Figure 1. Regression lines for HCM (FHW, 1997) and E18 Vestfold (Haugen, 1996). Data is from E18 Vestfold.

Figure 2. Time mean speed and space mean speed over time. E18 Vestfold. (Haugen, 1996)
4 Exercises

Here are some examples of how time mean speed and space mean speed can be calculated.

4.1 Exercise 1

The speeds of two vehicles were measured over a road section of 10 km. The first car was travelling at 60 km/h, and the second at 80 km/h. Both were travelling at a constant speed across the road section.

What is their average speed?

ANSWER:

**Time mean speed**

\[
\overline{u}_p = \frac{u_{p1} + u_{p2} + \ldots + u_{pn}}{n} = \frac{60 + 80}{2} = 70 \text{ km/h}
\]

**Space mean speed**

Space mean speed can be calculated in various ways, as shown below.

*Calculation based on average time duration*

The length of the road section is 10 km. Vehicle 1 is travelling at 60 km/h. Vehicle 2 is travelling at 80 km/h.

Thus, the time duration over 10 km will be:

\[
t_1 = \frac{d}{u_{s1}} = \frac{10\text{ km}}{60\text{ km/h}} = 0.167h = 10 \text{ min} = 600 \text{ sec}
\]

\[
t_2 = \frac{d}{u_{s2}} = \frac{10\text{ km}}{80\text{ km/h}} = 0.125h = 7.5 \text{ min} = 450 \text{ sec}
\]

\[
\bar{t} = \frac{t_1 + t_2 + \ldots + t_n}{n} = \frac{600 \text{ sec} + 450 \text{ sec}}{2} = 525 \text{ sec} = 8.75 \text{ min} = 0.1458h
\]

\[
\overline{u}_s = \frac{d}{\bar{t}} = \frac{10 \text{ km}}{0.1458h} = 68.57 \text{ km/h}
\]

*Calculation of space mean speeds using harmonic mean*

Vehicle 1 is travelling at 60 km/h. Vehicle 2 is travelling at 80 km/h.

\[
\overline{u}_s = \frac{\sum u_i}{n} = \frac{1}{\frac{1}{u_{s1}} + \frac{1}{u_{s2}}} = \frac{1}{\frac{1}{60} + \frac{1}{80}} = 68.57 \text{ km/h}
\]

N.B. The difference will be greater at lower speeds.

10 km/h and 30 km/h give \(\overline{u}_p = 20 \text{ km/h}\) and \(\overline{u}_s = 15 \text{ km/h}\).
4.2 Exercise 2

Assume a road section of 20 km. You are driving at 60 km/h for the first 10 km.

How fast do you have to drive for the next 10 km to achieve an average speed of 80 km/h?

**ANSWER:**

This problem can be solved in a number of ways. Two options are shown below:

- **Option 1:** Logically by considering time duration
- **Option 2:** As a mathematical equation by using a harmonic mean formula
- The answer in both cases will be 120 km/h

### Calculation based on logic and time duration

The length of the road section is 20 km.
You are driving at 80 km/h for 20 km, which takes 15 minutes.

You are driving at 60 km/h for the first 10 km. This takes 10 minutes. Therefore, you have 5 minutes left for the remaining 10 km. (5 min. = 1/12 h = 0.0833 h)

Thus, the speed over the last 10 km:

\[ u_s = \frac{d}{t} = \frac{10 \text{ km}}{1/12 \text{ h}} = 120 \text{ km/h} \]

### Calculation using formula for harmonic mean

Here, the following formula is used:

\[ \bar{u}_s = \frac{1}{\frac{1}{n} \sum_{i=1}^{n} u_i} = \frac{1}{\frac{1}{n} \sum_{i=1}^{n} \frac{d}{u_i}} = \frac{1}{\frac{1}{n} \sum_{i=1}^{n} \frac{1}{u_i}} \]

Travelling for 10 km at 60 km/h and then for 10 km at \( x \) km/h. Therefore \( d \)=10 km for both speeds, which means we can ignore \( d \) in our calculation. The average space mean speed is given as 80 km/h.

\[ \bar{u}_s = \frac{1}{\frac{1}{n} \sum_{i=1}^{n} \frac{1}{u_i}} = \frac{1}{\frac{1}{2} \left( \frac{1}{60} + \frac{1}{x} \right)} = 80 \text{ km/h} \]

By solving this equation, we get \( x \)=120 km/h.

By comparison:
100 km/h for the last 10 km will give an average speed of 75 km/h
110 km/h for the last 10 km will give an average speed of 77.65 km/h
4.3 **Exercise 3**

Registrations of speeds over 2 hours are made at a given point, and you have received the following data:

<table>
<thead>
<tr>
<th>Time period</th>
<th>No. of vehicles</th>
<th>Time mean speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:00–11:00</td>
<td>100</td>
<td>80 km/h</td>
</tr>
<tr>
<td>11:00–12:00</td>
<td>50</td>
<td>60 km/h</td>
</tr>
</tbody>
</table>

What is the time mean speed over the 2 hours?
What if they were space mean speeds instead of time mean speeds?

**ANSWER:**

**Time mean speed**

Here we have 100 vehicles with an average speed of 80 km/h for the first hour, and 50 vehicles with an average speed of 60 km/h for the second hour.

The average for the two hours will therefore be:

\[
\bar{u}_p = \frac{u_{p1} + u_{p2} + \ldots + u_{pn}}{n} = \frac{100 \cdot 80 + 50 \cdot 60}{100 + 50} = 73.33 \text{km/h}
\]

**Space mean speed**

Space mean speed can be calculated in various ways, as shown below. Here no road section length is given, but as we want to make a calculation using time duration, we can assume a road section length, calculate the average time duration and then convert it back to space mean speed. We can also use a harmonic mean formula and calculate the space mean speed using individual speeds. Both methods are shown below.
**Calculation using average time duration**

Assume a road section length of 10 km.
100 vehicles travel at 80 km/h and 50 vehicles at 60 km/h.

Thus, the time duration over 10 km will be:

\[
\begin{align*}
t_{60} &= \frac{d}{u_s} = \frac{10\text{km}}{60\text{km/h}} = 0.167\text{h} = 10\text{ min} = 600\text{sec} \\
t_{80} &= \frac{d}{u_s} = \frac{10\text{km}}{80\text{km/h}} = 0.125\text{h} = 7.5\text{ min} = 450\text{sec} \\
\bar{t} &= \frac{t_1 + t_2 + \ldots + t_n}{n} = \frac{100 \cdot 450\text{sec} + 50 \cdot 600\text{sec}}{100 + 50} = 500\text{sec} = 0.1389\text{h} \\
\bar{u}_s &= \frac{d}{\bar{t}} = \frac{10\text{km}}{0.1389\text{h}} = 72\text{km/h}
\end{align*}
\]

**Calculation using harmonic mean of point speeds**

100 vehicles travel at 80 km/h and 50 vehicles at 60 km/h.

\[
\bar{u}_s = \frac{1}{\frac{1}{\frac{1}{n}} + \frac{1}{\bar{u}_i}} = \frac{1}{\frac{1}{100+50} \left(\frac{1}{100} + \frac{1}{80} + \frac{1}{60}\right)} = 72\text{km/h}
\]
4.4 Exercise 4

The road section is 120 km long. Vehicle A travels at 60 km/h for the whole length of the road section. Vehicle B travels at 40 km/h for the first half of the section and then accelerates to 80 km/h. Which vehicle will reach the end of the road section first?

The length of the road section is 120 km. Vehicle A travels at 60 km/h and takes 2 hours.

Vehicle B travels at 40 km/h for the first 60 km. This takes 90 minutes. Vehicle B travels at 80 km/h for 60 km. This takes 45 minutes. Vehicle B therefore takes 2 hours and 15 minutes. In other words, vehicle A will reach the end of the road section first.

Time duration can be calculated with this formula:
\[ t = \frac{d}{u_s} \]

What speed does vehicle B have to travel at the second half of the road section in order to reach the end of the section at the same time as vehicle A?

The length of the road section is 120 km. Vehicle A travels at 60 km/h and takes 2 hours.

Vehicle B then travels at 40 km/h for 60 km, which takes 90 minutes. Vehicle B therefore has 30 minutes (0.5 hours) left for the last 60 km in order to reach the end of the section at the same time as vehicle A.

The speed for the last 60 km will therefore be:
\[ u_s = \frac{d}{t} = \frac{60 \text{ km}}{0.5 \text{ h}} = 120 \text{ km/h} \]
How can it be that vehicle B travels for the same amount of time at 40 km/h and 80 km/h?

The length of the road section is 120 km. Vehicle A travels at 60 km/h for and takes 2 hours.

Since vehicle B travels for an equal amount of time at 40 km/h and 80 km/h, we get 1 hour at each speed. Vehicle B travels at 40 km + 80 km in 2 hours, and therefore reaches the end of the section at the same time as vehicle A.

This can also be solved mathematically. The distance \( d \) is 120 km and the time for each of the speeds is the same \( t_1 = t_2 = t \).

\[
d = t_1 \cdot u_1 + t_2 \cdot u_2 = t \cdot u_1 + t \cdot u_2 = t \cdot (u_1 + u_2)
\]

\[
t = \frac{d}{u_1 + u_2} = \frac{120 \text{ km}}{40 \text{ km/h} + 80 \text{ km/h}} = 1 \text{ hour}
\]

Therefore, vehicle B travels for the same amount of time at 40 km/h and 80 km/h, which takes 2 hours over 120 km.

In other words, the same amount of time as vehicle A.
5  A theoretical perspective on average speeds with illustrative examples

A certain amount of total vehicle mileage\(^2\) is done at speed \(u\), along with a corresponding amount of travel time. These amounts can be described as statistical distributions, where the average speed can be interpreted as the expected value for each distribution. Time mean speed (TMS) can therefore be regarded as the mean value of the total vehicle mileage distribution of the speeds, while space mean speed (SMS) is the mean value of the travel time distribution of the speeds. Below we will look at the theory behind this result. With certain additions, we will follow the presentation in Danielsson (1999). Then we will show how the theoretical results can be applied in a real-world example.

5.1  Theoretical perspective

First, we will introduce a number of terms and definitions. Traffic is normally described as a phenomenon in both in time and space. Therefore, we assume that each point on the road network we are studying directly corresponds to a point \(x\) in the interval \([0, X]\), where \(X\) is the road network’s total length. The road network is examined during a time interval \([0, T]\), and \(t\) represents a point in time in this interval. The total area \([0, X] \times [0, T]\) is called \(A\). Figure 3 shows \(A\).

![Figure 3. Illustration of traffic distribution in time and space, \(A = [0, X] \times [0, T]\).](image)

**Flow.** By flow, we mean the number of vehicles that pass a certain point \(x\) on the road during period \([0, T]\). Flow is represented by \(q(x; T)\).

**Concentration.** By concentration (or density), we mean the number of vehicles on the road \([0, X]\) at any instant \(t\). Concentration is represented by \(r(t; X)\).

\(^2\) Vehicle mileage is measured as kilometres driven
Total vehicle mileage in area $A$ is defined as

$$Q(A) = \int_0^X q(x; T)dx$$

Total vehicle mileage is illustrated in Figure 4. The points on the horizontal axis represent crossroads or junctions where the flow increases or decreases. Total vehicle mileage is thus the integral of flow function $q(x; T)$ over interval $[0, X]$ or, alternatively, the green area in Figure 4.

![Figure 4. Total vehicle mileage over period $T$ on a hypothetical road network of length $X$](image)

Travel time in $A$ is defined as

$$R(A) = \int_0^T r(t; X)dt$$

This function is illustrated in Figure 5. The concentration function $r(t; X)$ changes over time when vehicles join or leave the road. The travel time taken to produce the total vehicle mileage $Q(A)$ is the integral of the concentration function over interval $[0, T]$ or, alternatively, the red area in Figure 5.

![Figure 5. Travel time on a hypothetical road network of length $X$ over period $T$](image)

The mean speed $\mu(A)$ can now be defined as

$$\mu(A) = \frac{Q(A)}{R(A)}$$

(1)
Note that $Q(A)$ is the total distance covered by all traffic over the time period, and $R(A)$ is the travel time used for this. The mean speed is therefore defined in terms of “distance divided by time”. This conforms with the usual physical definition of speed, the difference here being that distance and travel time are aggregated for many vehicles.

In order to describe speed distributions over both total vehicle mileage and travel time, we also need an expression of the flow and concentration at speed $u$. They are represented by $q(x, u; T)$ and $r(t, u; X)$ respectively. Total vehicle mileage in $A$ at speed $u$ can thus be expressed as

$$Q(u; A) = \int_0^X q(x, u; T) dx$$

and the travel time in $A$ at speed $u$ can be expressed as

$$R(u; A) = \int_0^T r(t, u; X) dt$$

For total vehicle mileage and travel time at a constant speed $u$, we have the relationship

$$Q(u; A) = u \cdot R(u; A)$$

i.e. distance equals speed multiplied by travel time. This is often called the fundamental identity. This then gives the relationship

$$Q(A) = \int_0^\infty Q(u; A) du = \int_0^\infty u \cdot R(u; A) du$$

Mean speed can thus be expressed as

$$\mu(A) = \frac{\int_0^\infty u \cdot R(u; A) du}{R(A)} = \int_0^\infty u \cdot \frac{R(u; A)}{R(A)} du$$

(2)

Mean speed can therefore be interpreted as the expected value of the travel time distribution of the speeds. This is equivalent to space mean speed.

The total vehicle mileage distribution of the speeds has of course also an expected value. This is the time mean speed and can be expressed as

$$\mu_Q = \int_0^\infty u \cdot \frac{Q(u; A)}{Q(A)} du$$

(3)

Danielsson (1999) points out that this expected value is in some sense a mean speed but has no single practical interpretation which distinguishes it from $\mu(A)$, which can be regarded as mean speed for the traffic travelling on the road. This can be understood since $\mu(A)$ according to expression (2) is the same as $\mu(A)$ according to (1), which is an interpretable parameter, while $\mu_Q$ according to (3) cannot be expressed in a similar easy way as expression (1).
Also note that \( \mu(A) \) can be expressed in the following way

\[
\mu(A) = \frac{1}{\int_{0}^{\infty} \frac{1}{u} \frac{Q(u; A)}{Q(A)} du}
\]

i.e. the harmonic mean value (of the vehicle mileage distribution of the speeds). This is useful when \( \mu(A) \) is estimated in real-world traffic measurements. (Which agrees with the result in section 2.2.).

5.2 Example

Below we illustrate how both time mean speed and space mean speed can be calculated using flow and speed measurements from one point on the road network. Here we follow the presentation Wardrop (1952). Speed is a continuous variable, but we can simplify things by using grouped data, where speeds are divided into groups of 5 km/h, in which all vehicles maintain a constant speed that is equal to the class midpoint. We also simplify the notation in the following way.

Assume that on road section \([0, X]\), we have \( C \) vehicle flows \( q_1, q_2, ..., q_C \) with the speeds \( u_1, u_2, ..., u_C \) and a total flow of \( Q = q_1 + q_2 + \cdots + q_C = \sum_{i=1}^{C} q_i \) and let \( f_i = q_i/Q \) for \( i = 1, 2, ..., C \). The values \( f_i \) constitute the total vehicle mileage distribution of the speeds, and using them we can calculate time mean speed as

\[
\bar{u}_p = \sum_{i=1}^{C} f_i \cdot u_i
\]

Consider now a partial flow \( i \) with flow \( q_i \) and speed \( u_i \). The average time interval between vehicles in the flow is clearly \( 1/q_i \), and the distance covered over this time period is \( u_i/q_i \). It follows that the concentration, i.e. the number of vehicles on the road section at any given instant, is \( k_i = q_i/u_i \). To understand this, we can use an example. Assume a flow of 100 cars that, over one hour, cover a distance of 1 km at 60 km/h. The average time interval between cars is 1/100 h (or 36 seconds). The distance between a given car and the car behind it is \( \frac{60}{q_i} = 0.6 \) km, i.e. 600 metres. The concentration at a given point in time is therefore \( k_i = \frac{100}{u_i} = 1.7 \), i.e. 1.7 cars on the road section.

The quantities \( k_1, k_2, ..., k_C \) therefore represent the concentrations of vehicles in the various partial flows and the total concentration is:

\[
K = \sum_{i=1}^{C} k_i
\]

By forming the weights \( f_i' = k_i/K \), we can then calculate space mean speed as
\[ \bar{u}_s = \sum_{i=1}^{c} f'_i \cdot u_i \]

Remark: The amount of concentration in partial flow \( i \) (speed group \( i \)), \( f'_i \), is also equal to the amount of travel time in partial flow \( i \). In Appendix 1, we demonstrate that this definition of SMS, which is expressed as a weighted mean with weights of concentrations (\( f'_i = k_i / K \)), is identical to the harmonic mean value of vehicle speeds (which can also be assumed to be constant over the interval \([0, X]\)), according to section 2.2.

In Table 1, artificial data from a hypothetical crossroads on a road with a speed limit of 70 km/h, where information has been collected about vehicle flow in various speed groups, is reported. All vehicle speeds in a group are assumed to have constant speed equal to the class midpoint.

<table>
<thead>
<tr>
<th>Speed range</th>
<th>Flow</th>
<th>Percentage in time</th>
<th>Weighted mean</th>
<th>Concentration</th>
<th>Percentage in space</th>
<th>Weighted mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>40–44(^3)</td>
<td>42</td>
<td>5</td>
<td>2.1%</td>
<td>0.9</td>
<td>0.119</td>
<td>3.2</td>
</tr>
<tr>
<td>45–49</td>
<td>47</td>
<td>1</td>
<td>0.4%</td>
<td>0.2</td>
<td>0.021</td>
<td>0.6%</td>
</tr>
<tr>
<td>50–54</td>
<td>52</td>
<td>20</td>
<td>8.4%</td>
<td>4.4</td>
<td>0.385</td>
<td>10.3%</td>
</tr>
<tr>
<td>55–59</td>
<td>57</td>
<td>41</td>
<td>17.2%</td>
<td>9.8</td>
<td>0.719</td>
<td>19.2%</td>
</tr>
<tr>
<td>60–64</td>
<td>62</td>
<td>57</td>
<td>23.9%</td>
<td>14.8</td>
<td>0.919</td>
<td>24.6%</td>
</tr>
<tr>
<td>65–69</td>
<td>67</td>
<td>38</td>
<td>16.0%</td>
<td>10.7</td>
<td>0.567</td>
<td>15.2%</td>
</tr>
<tr>
<td>70–74</td>
<td>72</td>
<td>42</td>
<td>17.6%</td>
<td>12.7</td>
<td>0.583</td>
<td>15.6%</td>
</tr>
<tr>
<td>75–79</td>
<td>77</td>
<td>21</td>
<td>8.8%</td>
<td>6.8</td>
<td>0.273</td>
<td>7.3%</td>
</tr>
<tr>
<td>80–84</td>
<td>82</td>
<td>9</td>
<td>3.8%</td>
<td>3.1</td>
<td>0.110</td>
<td>2.9%</td>
</tr>
<tr>
<td>85–90</td>
<td>87</td>
<td>4</td>
<td>1.7%</td>
<td>1.5</td>
<td>0.046</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

Total \( Q = 238 \) 100.0%  TMS = 64.9 \( K = 3.743 \) 100.0% SMS = 63.6

It is clear from the weighted mean columns that TMS = 64.9 km/h and SMS = 63.6 km/h. The value 63.6 km/h is also obtained by calculating the harmonic mean value of the 238 vehicles’ speeds.

Figures 6 and 7 show the time and space distribution of speed respectively. We can see that the space distribution (travel time) is slightly pushed to the left compared to the time distribution (vehicle mileage).

\(^3\)Group is strict 39.5 < \( u_i < 44.5 \)
The difference between $\bar{u}_p$ and $\bar{u}_s$ according to the above is therefore how the speeds $u_i$ are weighted. We can see that the terms time mean speed and space mean speed are derived from these weights. So we can argue:
• SMS: The weights $f_i'$ can be viewed as spatial. For a given road section $X$, we can study the travel time over the road section for vehicles of different speeds. The speed affects the length of time a vehicle is in space (road section). The travel times form a distribution as in figure 7. The weights $f_i'$ express this. This was also demonstrated mathematically in expression (2) above.

• TMS: The weights $f_i$ can be viewed as temporal. For a given length of time $T$, we can study the length of the road section. Since the length of the road section is the same for all vehicles, the total vehicle mileage across the road section is multiplied by the number of vehicles and their respective speed. In Figure 6, the distribution of total vehicle mileage can be seen. The weighs $f_i$ express this. This was also demonstrated mathematically in expression (3) above.

The terms time and space can therefore be said to depend on which perspective we want to measure speeds from.

5.3 **The link between vehicle mileage and flow and the link between travel time and concentration**

In Table 1, we are assuming that the measurements were made on a road section of known length. If the length of the road section is given, the vehicle mileage will only vary with the flow. The relative distribution of the flow will be equal to the relative distribution of the vehicle mileage.

In the same way, we can show that the relative distribution of concentration is the same as the relative distribution of travel time. In order to understand this, we need to understand the link between concentration and travel time. An illustration is provided below to help our understanding.

In Figure 5, a suggestion of how concentration can be illustrated is shown. The concentration on a road section is an instantaneous variable, and by measuring the concentration over a given time period, the travel time can be determined. In Figure 5, let’s assume that at time point 0, there are 10 vehicles on the road section. Let’s imagine we are taking a satellite photo of the road section and calculate the number of vehicles on the section. The height of the column in Figure 5 is therefore 10 on the y-axis at time point $t = 0$. Assume we take another photo 1 second after $t = 0$ (if we measure $t$ in seconds, we have $t = 1$), if there are still 10 cars on the road section, the concentration at that point in time is also 10. Since we have now moved to the right along the x-axis in Figure 5, we have an area under the curve. The area is $1 \times 10 = 10$ seconds, and this is the travel time for the 10 cars over a time interval of 1 second. We continue taking satellite photo of the road section every single second. Assume that at second 9, another car enters the road section. This means the concentration increased to 11 cars. In Figure 5, this is visible by the fact that the column rises by 1 car at second 9. Let’s also assume there are 11 cars on the road section, and in the 6 seconds following that another

---

4 From a statistical point of view, we could say we condition on the road section. SMS is the conditional mean value for spatially weighted speeds.
car enters the road section. The travel time for the 10 cars from seconds 1–9 is therefore \(9 \times 10 = 90\), and \(6 \times 11 = 66\) for the 11 cars over the last 6 seconds, meaning the total travel time is \(90 + 66 = 156\) seconds. In a similar way, we can progress through the whole time period \(T\) we want to study (for example an hour or a 24-hour period). The unit we usually measure in is kilometres for a road section and \(h\) for time (not in seconds as in the example above).

It is explained above that the concentration can be calculated by \(k_i = q_i / u_i\), which is also illustrated in Table 1. For the 40–44 km/h group, this gives \(k_i = 0.119\), can this also be interpreted as a travel time? Yes, in fact it can, but it applies to a unit distance. This might need to be clarified. In Table 2, some of columns are the same as in Table 1, but with the addition of the three columns on the right. In the “One vehicle” column, travel time is calculated for a vehicle travelling at 42 km/h over 1 km. This is given as \(\frac{1}{42} = 0.0238\) \(h\), which is equivalent to 1 minute and 24 seconds. If five vehicles travel at this speed across the road section over this time period, their combined travel time will be \(0.0238 \times 5 = 0.119\) \(h\), which is the same as the concentration \(k_i = \frac{q_i}{u_i} = \frac{5}{42} = 0.119\).

<table>
<thead>
<tr>
<th>Speed range</th>
<th>Flow</th>
<th>Concentration</th>
<th>Percentage in space</th>
<th>Travel time in hours (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(u_i)</td>
<td>(q_i)</td>
<td>(k_i = q_i / u_i)</td>
<td>(f_i = k_i / K)</td>
</tr>
<tr>
<td>40–44</td>
<td>42</td>
<td>5</td>
<td>0.119</td>
<td>3.2</td>
</tr>
<tr>
<td>45–49</td>
<td>47</td>
<td>1</td>
<td>0.021</td>
<td>0.6%</td>
</tr>
<tr>
<td>50–54</td>
<td>52</td>
<td>20</td>
<td>0.385</td>
<td>10.3%</td>
</tr>
<tr>
<td>55–59</td>
<td>57</td>
<td>41</td>
<td>0.719</td>
<td>19.2%</td>
</tr>
<tr>
<td>60–64</td>
<td>62</td>
<td>57</td>
<td>0.919</td>
<td>24.6%</td>
</tr>
<tr>
<td>65–69</td>
<td>67</td>
<td>38</td>
<td>0.567</td>
<td>15.2%</td>
</tr>
<tr>
<td>70–74</td>
<td>72</td>
<td>42</td>
<td>0.583</td>
<td>15.6%</td>
</tr>
<tr>
<td>75–79</td>
<td>77</td>
<td>21</td>
<td>0.273</td>
<td>7.3%</td>
</tr>
<tr>
<td>80–84</td>
<td>82</td>
<td>9</td>
<td>0.110</td>
<td>2.9%</td>
</tr>
<tr>
<td>85–90</td>
<td>87</td>
<td>4</td>
<td>0.046</td>
<td>1.2%</td>
</tr>
<tr>
<td>Total</td>
<td>238</td>
<td>Q = 3.743</td>
<td>100.0%</td>
<td>Total 3.743</td>
</tr>
</tbody>
</table>

Table 3 uses the exact same basis as Table 2, but we are assuming that the road section being studied is 3.3 m long (like when laying tubes and using Metor 3000). The time one vehicle at 42 km/h needs to travel 3.3 m is \(\frac{3.3}{42} = 0.000079\) \(h\). For 5 vehicles, the combined time to travel the road section will be \(0.000079 \times 5 = 0.00039\) \(h\). This means the combined time duration needed for all 238 vehicles to travel a distance of 3.3 m is \(0.01235\) \(h\). In Table 2, where the distance was 1 km, the total time duration was 3.743 \(h\). It should now be clear that the method of calculating the concentration \(k_i = q_i / u_i\) requires a reference to a unit distance (1 km). Also note that the relative distribution of travel times is the same in Tables 2 and 3, and does not depend on the length of the road section.

\(5\) Km/h
Table 3. Link between concentration and travel time. Road section = 0.0033 km, i.e. 3.3 m.

<table>
<thead>
<tr>
<th>Speed range</th>
<th>$u_i$</th>
<th>$q_i$</th>
<th>$k_i = q_i/u_i$</th>
<th>$j_i^t = k_i/K$</th>
<th>One vehicle</th>
<th>All vehicles</th>
<th>Relative travel time (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40–44</td>
<td>42</td>
<td>5</td>
<td>0.119</td>
<td>3.2</td>
<td>0.000079</td>
<td>0.00039</td>
<td>3.2</td>
</tr>
<tr>
<td>45–49</td>
<td>47</td>
<td>1</td>
<td>0.021</td>
<td>0.6%</td>
<td>0.000070</td>
<td>0.000077</td>
<td>0.6%</td>
</tr>
<tr>
<td>50–54</td>
<td>52</td>
<td>20</td>
<td>0.385</td>
<td>10.3%</td>
<td>0.000063</td>
<td>0.000127</td>
<td>10.3%</td>
</tr>
<tr>
<td>55–59</td>
<td>57</td>
<td>41</td>
<td>0.719</td>
<td>19.2%</td>
<td>0.000058</td>
<td>0.000237</td>
<td>19.2%</td>
</tr>
<tr>
<td>60–64</td>
<td>62</td>
<td>57</td>
<td>0.919</td>
<td>24.6%</td>
<td>0.000053</td>
<td>0.000303</td>
<td>24.6%</td>
</tr>
<tr>
<td>65–69</td>
<td>67</td>
<td>38</td>
<td>0.567</td>
<td>15.2%</td>
<td>0.000049</td>
<td>0.000187</td>
<td>15.2%</td>
</tr>
<tr>
<td>70–74</td>
<td>72</td>
<td>42</td>
<td>0.583</td>
<td>15.6%</td>
<td>0.000046</td>
<td>0.000193</td>
<td>15.6%</td>
</tr>
<tr>
<td>75–79</td>
<td>77</td>
<td>21</td>
<td>0.273</td>
<td>7.3%</td>
<td>0.000043</td>
<td>0.000090</td>
<td>7.3%</td>
</tr>
<tr>
<td>80–84</td>
<td>82</td>
<td>9</td>
<td>0.110</td>
<td>2.9%</td>
<td>0.000038</td>
<td>0.000018</td>
<td>2.9%</td>
</tr>
<tr>
<td>85–90</td>
<td>87</td>
<td>4</td>
<td>0.046</td>
<td>1.2%</td>
<td></td>
<td>0.000036</td>
<td>1.2%</td>
</tr>
<tr>
<td>Total</td>
<td>238</td>
<td></td>
<td>3.743</td>
<td>100.0%</td>
<td></td>
<td>0.01235</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

5.4 Brief remark on the use of the term point speed

The physical definition of speed is distance divided by time, i.e. $u_i = \frac{d}{t_i}$, where $d = distance$ and $t_i = travel time$ for vehicle $i$ (see section 2.2). Assume that on a road section of length $d$, we are measuring the time taken to cover the section by every single one of $n$ vehicles that pass through the section. Thus TMS and SMS can be calculated in the following way

1. calculate mean value of the $u_i$ values according to $\bar{\mu}_p = \frac{\sum u_i}{n}$, which gives TMS
2. calculate mean value of the $t_i$ values according to $\bar{t} = \frac{\sum t_i}{n}$, and then $\bar{\mu}_s = \frac{d}{\bar{t}}$, which gives SMS (alternatively the harmonic mean value of the $u_i$ values can be calculated, which gives the same result).

Note that in these calculations, there is not any limit on the length of the road section. It could be $3.37$ m, $10$ m, $100$ m, $1$ km or any other totally arbitrary distance ($157.9$ m). Note that time mean speed (TMS) can thus be calculated over a longer road section. Time mean speed implies that it is (the distribution of) speeds in time (with a link to total vehicle mileage) which matters (as shown in Figure 6), and space mean speed implies that it is (the distribution of) speeds in space (with a link to travel time, see Figure 7) which matters.

---

6 Km/h
7 As in tube measurement with Metor 3000
6 Advantages and disadvantages of TMS and SMS

When choosing between TMS and SMS, there are a number of arguments for using SMS for speed studies.

- SMS estimates a parameter, $\mu(A)$, which has a clear interpretation as the average speed of the road traffic. What TMS estimates is more difficult to interpret.
- SMS uses the physical definition of speed: “distance divided by time” or “total vehicle mileage divided by total travel time”.
- The so-called fundamental identity $Q(u; A) = u \cdot R(u; A)$, i.e. the flow, is equal to the speed multiplied by the travel time only if the speed is measured with SMS.
- The time vehicles spend on successive road sections can be added together, whereas speeds cannot. SMS is based on travel times and therefore has an advantage when it comes to making accurate estimations. TMS, which is based on adding speeds together, therefore has a built-in problem structure.

On the other hand:

- The difference between TMS and SMS is often small if we are not concerned about congestion problems.
- Estimations of changes in speed, for example speed index, based on TMS and SMS are likely to be very similar.
- We may also be forced to choose one method over the other if the measurement equipment or previously collected data does not allow for calculation of both. The Swedish speed index, for example, uses data saved as arithmetical mean values of vehicle speeds per hour. SMS can thus not be calculated, but TMS should give a very similar result.
- Wardrop points out that measurements of travel time often have a greater variability than measurements of speed. This means that the precision of TMS is often greater than that of SMS.

There are clearly advantages and disadvantages of both SMS and TMS. Perhaps the most important thing is to stick to the same method when making repeated measurements.
7 Estimating space mean speed using occupancy

We do not always have the option of installing equipment for recording space mean speed. Since space mean speed is determined using induction loops, occupancy (or detector coverage level) can be used to estimate density, and thereby to estimate space mean speed (FHWA, 1992).

When devising formula frameworks, the formula for space mean speed (chapter 2.2) can be used along with Greenshield’s classic formula for the relationship between traffic flow, space mean speed and density of space mean speed (FHWA, 1992).

Formula for space mean speed from chapter 2.2:

\[
\overline{u}_s = \frac{\sum_{i=1}^{n} d}{\sum_{i=1}^{n} u_{si}} = \frac{\sum_{i=1}^{n} \frac{d}{u_{si}}}{\sum_{i=1}^{n} \frac{1}{u_{si}}}
\]

Where:
- \(\overline{u}_s\) = Space mean speed
- \(d\) = Length of road section
- \(u_{si}\) = Travel speed of vehicle \(i\)
- \(t_i\) = Travel time for vehicle \(i\)
- \(n\) = Number of vehicles

Greenshield’s formula:

\[
q = \overline{u}_s \cdot k
\]

Where:
- \(\overline{u}_s\) = Space mean speed \([\text{km/h}]\)
- \(k\) = Density \([\text{veh./km}]\)
- \(q\) = Traffic flow \([\text{veh./km}]\)

And traffic flow can be expressed as:

\[
q = \frac{N}{T} \text{ where } N \text{ is the number of vehicles and } T \text{ is the time period}
\]
Density can also be estimated using detector coverage level (occupancy), i.e. the amount of time a detector is covered (occupied). In practice, this means the amount of time a vehicle spends over a detector. Figure 8 attempts to demonstrate this visually. The detector has length $L_d$ and the vehicle has length $L_i$. The detector is covered from the time the front of the vehicle passes over the detector until the rear end of the vehicle leaves the detector.

**Detector coverage level (occupancy) can thus be defined as:**

\[
\text{occupied time } T_{occ} = \frac{L}{T} \sum_{i} \frac{1}{u_i} + L_d \cdot k = \frac{L}{T} \left( \sum_{i} \frac{1}{u_i} + L_d \cdot k \right) = L \cdot q \cdot \frac{1}{\bar{u}_s} + L_d \cdot k = L \cdot k + L_d \cdot k
\]

If we assume that all vehicles have the same length ($L_i=L$), the formula can be simplified to:

Based on Greenshield’s formula, space mean speed can be estimated as:

\[
\bar{u}_s = \frac{q}{k} = \frac{q \cdot (L + L_d)}{occ}
\]
8 Other terminology

The terminology within the road traffic area is extensive. A reference with definitions is ITS Terminology (2018), where the terms are stated in five different languages. Below, we give some examples.

8.1 Travel time/journey time

Travel time is normally defined as the time spent between two defined points, including operating time, wait time and transfer time if relevant.

Journey time is used to calculate journey speed, while travel time is used to calculate space mean speed.

The term “driving time” is sometimes also used.

8.2 Undelayed travel time

Undelayed travel time is the time spent undisturbed by other traffic, or travel time without the effects of other traffic, i.e. travel time when traffic is in free flow. This travel time can also include delays due to regulation and roadworks, for example speed limits, geometry, road standards etc.

8.3 Delay

Delay is any additional travel time on top of undelayed travel time. Delay also refers to what we call traffic-dependent delays, i.e. delays due to other traffic.

8.4 Current travel time

Current travel time is the travel time at the present moment in time, i.e. the travel time recorded at this very moment.

8.5 Normal travel time

Normal travel time is the travel time we can normally expect on a road section at a given moment in time.

---

\(^8\) Finnish, Swedish, Norwegian, Danish and Icelandic.
It is also often the case that the term “normal travel time” is used when we actually mean “undelayed travel time”.
References

Danielsson, S. Statistiska metoder vid analys av trafiksäkerhet. Linköping University, Department of Mathematics. 1999.


Appendix 1

In Wardrop’s notation, SMS is given by $\bar{u}_s = \sum_{i=1}^{C} f_i' \cdot u_i$. According to section 2.2, the harmonic mean value of speeds $u_i$ is given by

$$\bar{u}_s = \frac{1}{n \sum_{i=1}^{n} \frac{1}{u_i}}$$

It is demonstrated below that $\bar{u}_s = \sum_{i=1}^{C} f_i' \cdot u_i$ can also be expressed as a harmonic mean value. Note first that Wardrop assumes grouped data with $C$ groups (flows). A harmonic mean value based on $n$ sets of individual speeds is given when grouping into $C$ groups by

$$\bar{u}_s = \frac{1}{\sum_{i=1}^{n} \frac{1}{u_i}} = \frac{1}{\sum_{i=1}^{C} \frac{1}{u_i}} = \frac{1}{Q \sum_{i=1}^{C} k_i}$$

(4)

We have used the fact that $n = \sum_{i=1}^{n} 1 = \sum_{i=1}^{C} q_i = Q$ is the total flow. We have also used the fact that if we add $n$ sets of individual speeds $1/u_i$ together, we can group identical vehicle speeds for one set of grouped data. For example, if 4 vehicles have the same speed, we have $\frac{1}{u_i} + \frac{1}{u_i} + \frac{1}{u_i} + \frac{1}{u_i} = 4 \cdot \frac{1}{u_i} = q_i \cdot \frac{1}{u_i}$. Now rewrite $\bar{u}_s$ according to Wardrop’s notation

$$\bar{u}_s = \sum_{i=1}^{C} f_i' \cdot u_i = \sum_{i=1}^{C} \frac{k_i}{K} \cdot u_i = \frac{\sum_{i=1}^{C} q_i \cdot u_i}{K} = \frac{\sum_{i=1}^{C} q_i}{K} = \frac{Q}{K} = \frac{1}{Q \sum_{i=1}^{C} k_i}$$

(5)

The last equality in (5) is the same as the last equality in (4), which should be shown.

---

*The groupings could be slightly broader, e.g. 5 km/h, or narrower, e.g. 1 km/h, or much narrower, e.g. 0.01 km/h. The narrower the groupings are, the smaller rounding errors will be.*
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